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THE MEASUREMENT OF LARGE STRAINS WITH FOIL RESISTANCE STRAIN GAGES

Robert E. Franz

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US ARMY ARMAMENT RESEARCH AND DEVELOPMENT COMMAND
BALLISTIC RESEARCH LABORATORY
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Gage factor changes in foil resistance strain gages were observed and measured. Errors due to these changes are discussed and corrections for the gages studied were determined. Residual volume changes during plastic deformation in 7039 Aluminum Alloy and S-7 tool steel were shown to be small and unmeasurable with the gages used. The gage factor changes were assumed to be the result of plastic work in the gage material. Estimates of the magnitude of the resistance changes due to plastic deformation in constantan were made using tension experiments on wires.

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I. INTRODUCTION

Since their commercial introduction in the 1940's bonded resistance strain gages have been used with great success where strain or displacement of a surface is used as a measure of some physical quantity such as stress. Bonded strain gages were quickly accepted because of their superior sensitivity and especially their ease of use.

The first commercial gages were made of grids of fine metallic wire attached to paper backing bonded on the surface to be strained. In 1952 the first foil gages were introduced in England. These gages are made from a thin foil of a suitable metal, such as constantan, which is bonded to a thin paper or plastic backing and etched into a grid. Foil gages come in many shapes, sizes, and configurations.

For strains below a few tenths of a percent, that is, elastic strains in most metals, bonded strain gages were and are truly superior in sensitivity, linearity, reproducibility, and ease of use. The same can not be said for their use in measuring large plastic strains. Changes in gage constant and zero shifts in cyclic use are often seen. Nevertheless, their superior sensitivity and applicability for both static and dynamic measurement have resulted in resistance gages designed to measure large strains.

II. GAGES FOR LARGE STRAIN MEASUREMENT

The first resistance gages designed to measure large strains were made by Swainger[1] of Minalpha wire and Weibull[2] of Copel. Copel is a copper-nickel alloy (55 Cu, 45 Ni) which is close to constantan (60 Cu, 40 Ni) in constitution. The most popular resistance foil gages used for large plastic strains are made of constantan or Advance. Bridgman[3] showed that the pressure coefficient of resistance is essentially the same for Cu-Ni alloys of from 20-80% nickel. Kuczynski[4] also showed that the strain coefficient of resistivity of these alloys was constant over the range from 40-60% nickel and was not affected by wire size down to 0.025 mm diameter. Shoub[5] measured 0.025 mm diameter constantan wires to 22% strain and found a linear relationship with

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[1]*K. H. Swainger, "Electrical Resistance Strain-Gauges to Measure Large Strains," Nature, 159, 61-62, 1947.*

[2]*W. Weibull, "Electrical Resistance of Wires with Large Strains," Nature, 162, 966-67, 1948.*

[3]*P. W. Bridgman, "The Effect of Pressure on the Resistance of Three Series of Alloys," Proceedings of American Academy of Arts and Sciences, 63, 329-345, 1928.*

[4]*G. C. Kuczynski, "Effect of Elastic Strain on the Electrical Resistance of Metals," Physical Review, 94, 61-65, 1954.*

[5]*H. Shoub, "Wire-Resistance Gages for the Measurement of Large Strains," David Taylor Model Basin, Report No. 570, March 1950.*

a slope of 2.02 for $\ln R/R_o$ versus $\ln \ell/\ell_o$, where R is the resistance and ℓ the length of the wire. The zero subscript denotes their initial values. This is very close to Weibull's[2] logarithmic slope of 2.0, that is,

$$\frac{R}{R_o} = \left(\frac{\ell}{\ell_o}\right)^2 = (1+\varepsilon)^2. \tag{1}$$

Here $\frac{\ell}{\ell_o} = 1+\frac{\Delta\ell}{\ell_o}$ and $\varepsilon = \frac{\Delta\ell}{\ell_o}$ where $\Delta\ell$ is change in length and ε is the strain. This result would be expected if the resistivity and the volume of the wire do not change during plastic deformation.[2] Franz, Benck, and Diberardo[6] show this same result with high elongation foil gages made of constantan* compared with optical strain measurements. Noting that,

$$\frac{\Delta R}{R_o} = \frac{R}{R_o} - 1 = (2+\varepsilon)\ \varepsilon \tag{2}$$

and since

$$\frac{1}{\varepsilon}\frac{\Delta R}{R_o} = \text{Gage Factor}, \tag{3}$$

A gage factor of $2+\varepsilon$ should be used to calculate strains for this type gage.

Weibull[2] also ran high strain rate tests with rates to approximately 60/sec. (34% maximum strain). He found no change in the logarithmic slope. Hauver and Melani[7] have used foil strain gages made of constantan in impact tests at much higher rates and found the same gage factor of $2+\varepsilon$ when compared to optical strain measurements. These results and those in Reference 6 indicate that there is no problem with the gages tracking the substrate strain, since free wires and bonded gages give the same results. More will be said later about gage tracking.

In most cases, the nominal gage factors of well annealed constantan high elongation gages are within a few percent of 2. Thus, for large strain measurements where errors of a few percent of the total strain are acceptable, $2+\varepsilon$ can be used as the gage factor. If greater accuracy in measurement is needed, a closer look must be taken at the mechanism of the gage's piezoresistive response and its strain sensitivity. The next section demonstrates a situation in which this greater accuracy is needed.

**Micro-Measurements EP-08-062TT-120*

[6] *R. E. Franz, R. F. Benck, and D. A. Diberardo, "Quasi-Static Stress-Strain Curves, S-7 Tool Steel," ARBRL-MR-03067, Ballistic Research Laboratory, October 1980 (AD A093773).*

[7] *G. E. Hauver and A. Melani, "Strain-Gage Techniques for Studies of Projectiles During Penetration," ARBRL-MR-03082, Ballistic Research Laboratory, February 1981 (AD A098660).*

III. VOLUME CHANGES DURING PLASTIC DEFORMATION

For some time this laboratory has used bonded foil resistance strain gages to measure strains during uniaxial tension and compression tests. In these tests both axial and circumferential strains are measured with gages bonded directly to the specimen. True stress and Poisson's ratio can then be calculated. Great care is taken, especially in compression tests, to preclude friction and bending in samples that have a 3:1 length to diameter ratio.[6] Resistance data are logged directly and are measured with a four terminal method to 3 milliohm accuracy and one milliohm sensitivity. Careful reduction of these data include corrections for transverse sensitivity and the effect of large strain as previously described.

Using these results, the volume change during and after deformation can be calculated. Surprisingly, many materials apparently showed what were considered large, permanent, positive volume changes in both tension and compression. Some changes as large as 0.2% were calculated.

These changes were large enough that, although they depended on the difference of two small numbers, the estimated errors in measurement were supposedly less than the effect by at least a factor of ten. They were also large enough to be measured directly by careful liquid displacement experiments to determine the density of the samples before and after deformation.

Therefore, density measurements were made on steel specimens plastically deformed in compression and aluminum alloys deformed in both tension and compression.

The measurements were made with a single pan analytic balance having a sensitivity of 0.01 milligram. Water was used as the immersion liquid and was kept a few degrees above room temperature by a circulating water bath controlled to 0.01°C. The suspension wire was made of nichrome prepared by a method devised by Bowman and Schoonover[8] to minimize miniscus problems.

Individual density runs of around five measurements on a single sample showed standard deviations of 100-200 ppm. Repeat runs on the same samples gave inter-run deviations of 200 ppm.

The experiments did not confirm the magnitude of the strain gage measurements but showed very small increases in volume or no volume change within the measurement error. Table I shows typical runs of data for 7039 aluminum alloy in compression. These results indicated that the strain gage data was in error and experiments were undertaken to simultaneously measure the strain mechanically.

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[8] *H. A. Bowman and R. M. Schoonover, "Procedure for High Precision Density Determinations by Hydrostatic Weighing," Journal of Research of National Bureau of Standards, 71C, No. 3, July-August 1967.*

TABLE I

UNIAXIAL COMPRESSION TESTS OF 7039 ALUMINUM ALLOY

TEST NO.	INITIAL DENSITY KG/M^3	FINAL DENSITY KG/M^3	PLASTIC STRAIN %	VOLUMETRIC STRAIN %
115	2740.470	2740.112	2.6	.013
143	2749.346	2748.766	2.6	.021
144	2738.044	2737.128	2.8	.033
122	2740.626	2739.961	3.8	.024
117	2740.585	2739.678	6.0	.033

Note: All volumetric strains are within measurement errors.

IV. MECHANICAL AND ELECTROMECHANICAL MEASUREMENTS OF STRAIN

Careful diametral measurements on samples loaded in uniaxial stress were made with a system similar to an old design by Peterson and Wahl.[9] The mechanical displacement measurements are made with a Huggenberger displacement extensometer gage.[10] Also attached was a Unimeasure 80,* a semiconductor resistance displacement transducer. Simultaneous strain gage measurements were made using two 90° rosette strain gages of the type used in the other tests.**

The gages were mounted on 6.35 mm diameter samples 180° apart on the axis 90° from the mechanical measurement.

Figure 1 shows a photograph of the system mounted in the testing machine. The Huggenberger gage has a sensitivity of 0.5 micrometer. The Unimeasure 80 used with a high resolution digital multimeter gave a sensitivity of 0.05 micrometer. Accuracy of the electromechanical measurements were estimated as 5%. Deviation from linearity of the measurements over the 0.05 mm displacement range was less than 1%.

**Unimeasure, Inc., Grants Pass, OR 97526.*

***Micro-Measurements EP-08-062TT-120.*

[9] *R. E. Peterson and A. M. Wahl, "Fatigue of Shafts at Fitted Members, with a Related Photoelastic Analysis," Journal of Applied Mechanics, Trans. of ASME, 57, 1935 (A-1).*

[10] *See for example "Handbook of Experimental Stress Analysis," M. Hetényi, ed., John Wiley & Sons, New York, pp 94-96, 1950.*

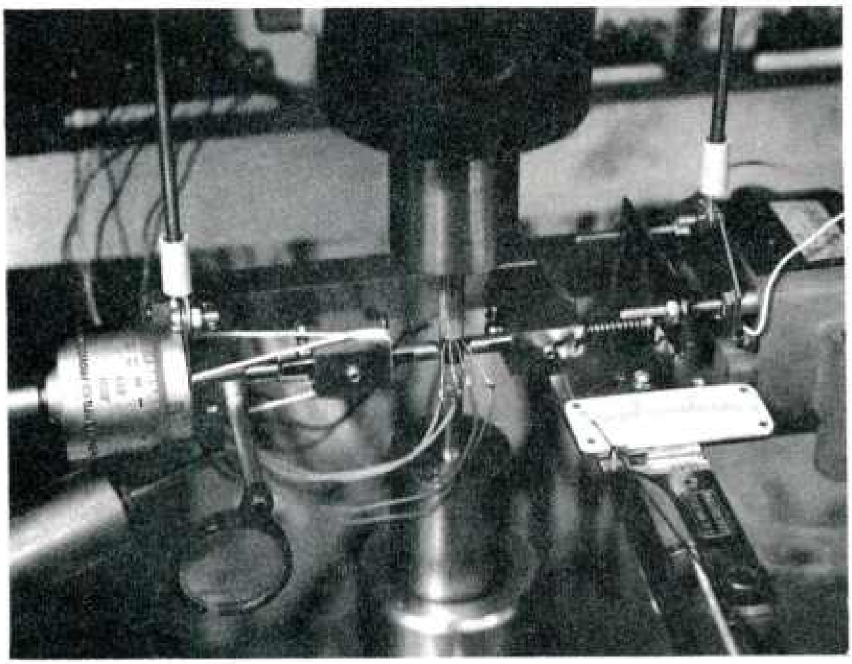

Figure 1. System for Measuring Diametral Strain and Displacement.

Figure 2 shows some results from tension tests using berylium-copper specimens which have a high elastic yield strain (>0.6%). The Unimeasure 80 strains are plotted against the strain gage results using the usual data reduction methods previously described. The Huggenberger and the Unimeasure 80 data showed excellent agreement. See Figure 3.

Figure 4 shows the results of a compression test on steel.

It is evident that the response of the strain gages changed when the strain in the specimen exceeded a few tenths of a percent. A gage in tension showed an increase in gage factor, one in compression a decrease. These changes were measured to be 2-3% in both tension and compression for the gages used.

In general, these small changes would not be seen since the gage factors of commercial strain gages are usually determined using small strains within the gage material's elastic range or slightly above. Conversely, when Weibull was measuring the response of wires to large plastic strain, his range was too large and small changes at yield were undetected. Weibull's first data point was at 5% strain. The yield strain of annealed Copel is probably less than 0.15%.

V. DISCUSSION

It was clear that the measured residual volume increases were due to the gage factor changes measured. These changes occur at the elastic-plastic yield of the gage material and are probably caused by plastic work. Plastic work increases the resistance of the gage in both tension and compression. For most materials the plastic work in uniaxial stress is almost linear with plastic strain so that the change in gage factor is a constant. See Appendix A for a more detailed explanation of gage response.

The fact that no measurable (with strain gages) volume change occurs can be used to determine the change in gage factor that occurs at the yield. That is, we can choose a gage factor change which will correct the volume data to show no residual volume change. The gage factor, of course, reverts to its original value when unloading occurs since no additional plastic work occurs. This leads to another consideration which must be made since the gages were shown to track the substrate. The stress in the substrate can be very much higher than that in the gage material. This is true especially for steels and other high strength alloys. This means that the elastic unloading strain in the substrate can be much higher than the elastic strain in the gage material. In fact, in some cases the gage material is not only unloaded but reloaded past its yield point in the reverse direction. This means that the gage factor once again changes but in the opposite direction.

The gages' yield strain was determined from the electromechanical measurements as 0.14%. The reverse loading yield strain on unloading was taken as twice the original yield or 0.28%. That is, after the substrate had unloaded 0.14%, the gage factor was changed.

The final strain determination analysis consisted of a computer program which calculated the strains using the appropriate gage factors and original

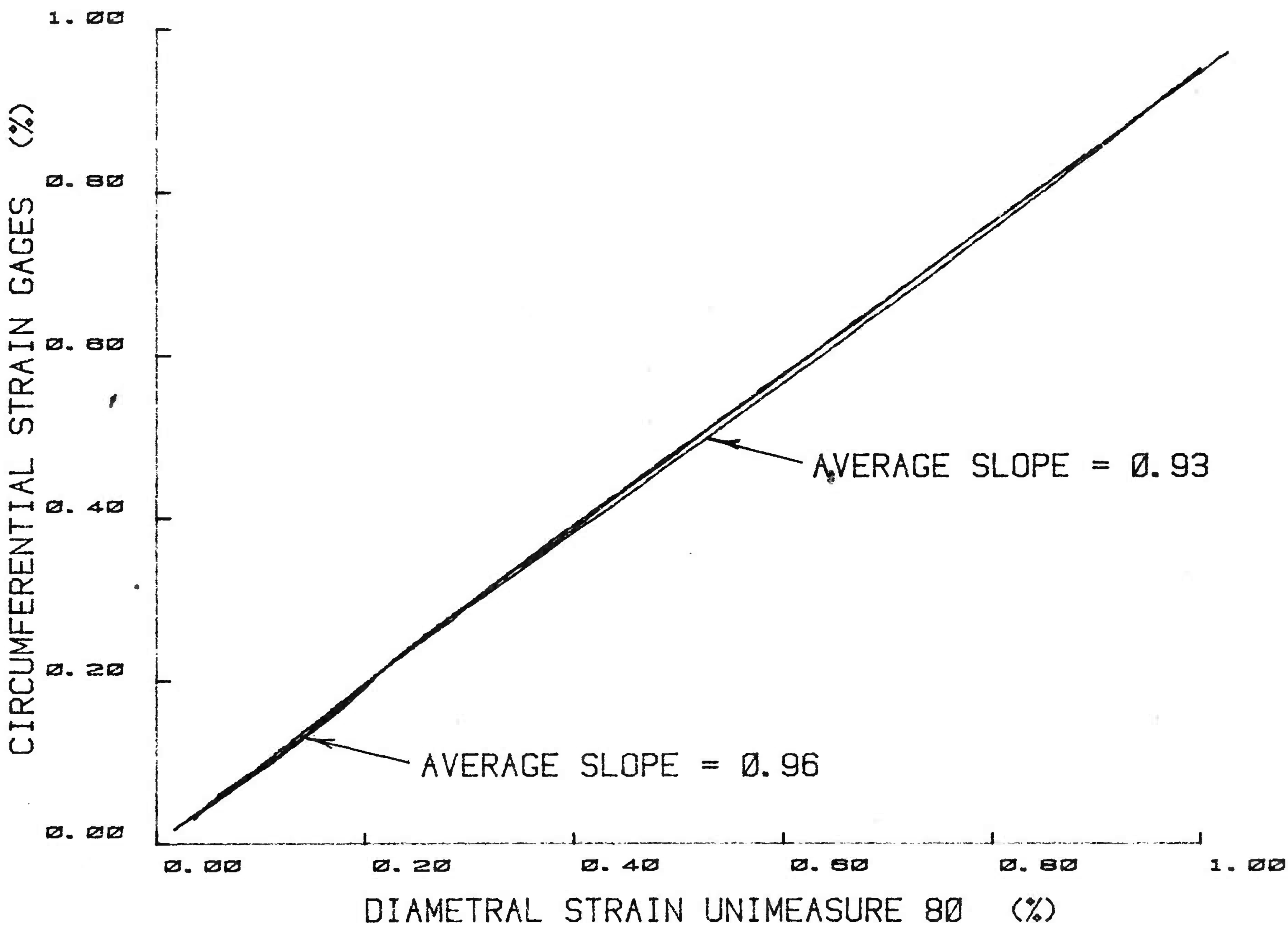

Figure 2. Three Uniaxial Tension Experiments on Berylium-Copper.

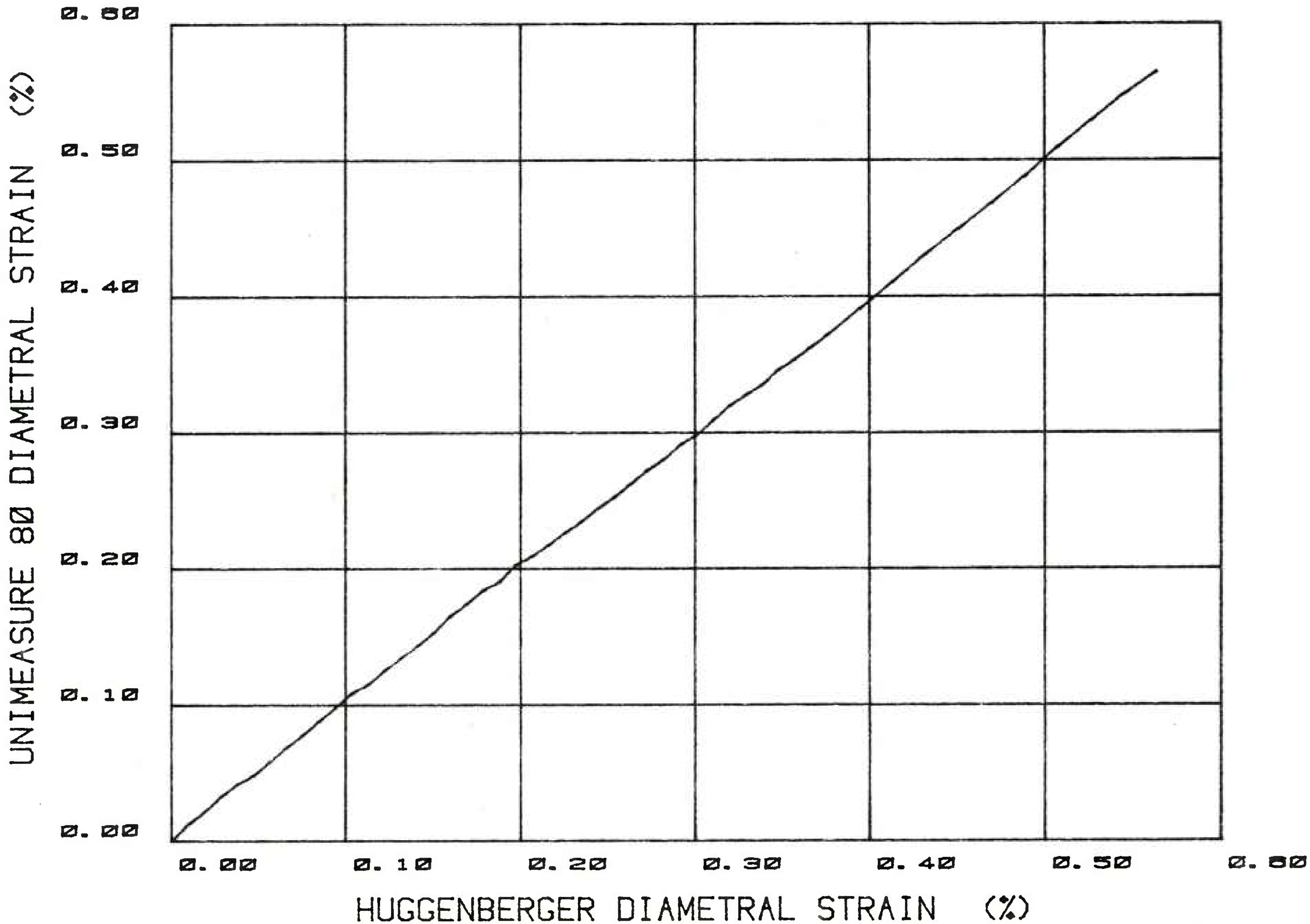

Figure 3. Diametral Strain Measured Using Unimeasure 80 vs. That Measured Using a Huggenberger Extensometer.

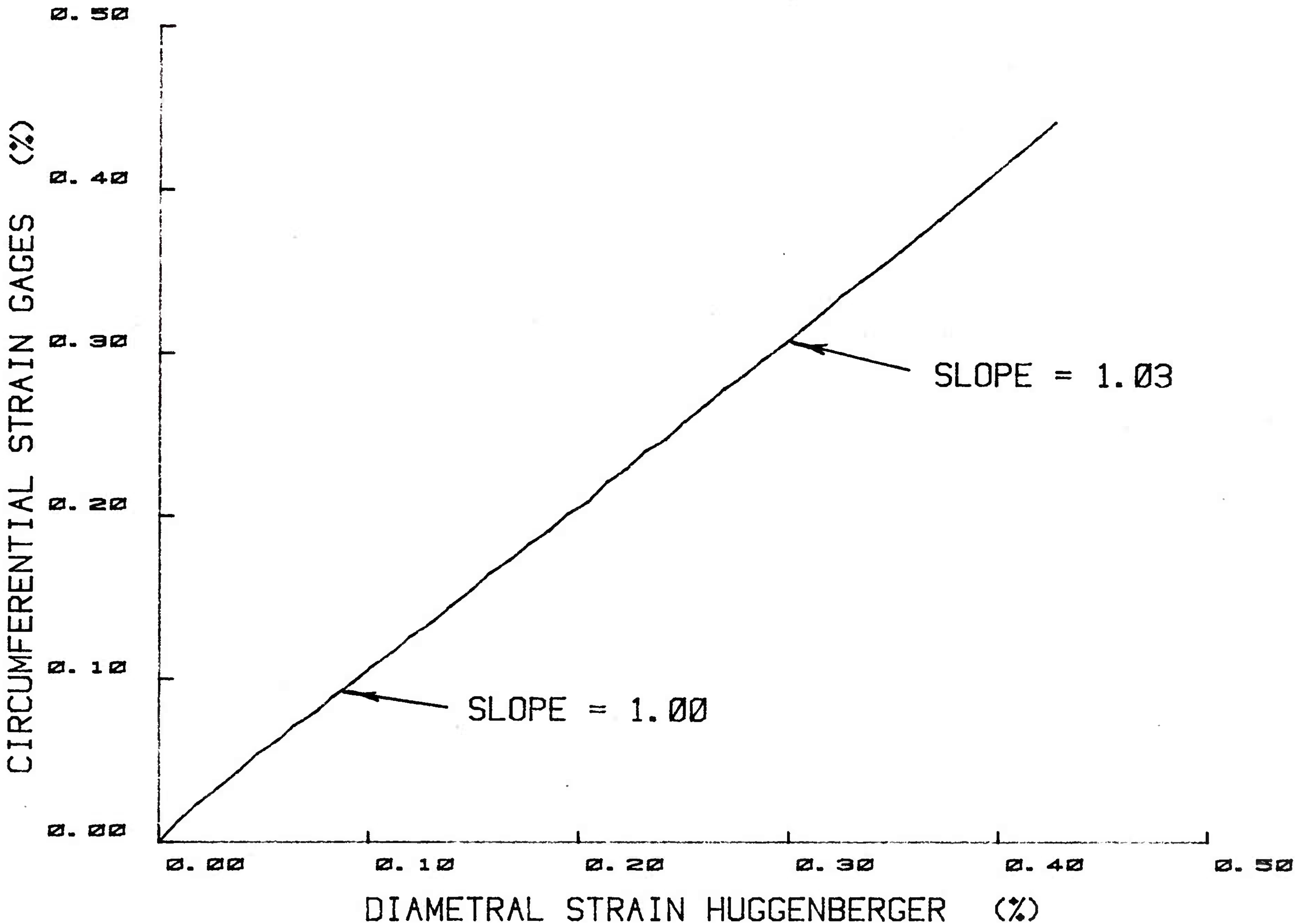

Figure 4. Uniaxial Compression Test on Rolled Homogeneous Armor Steel.

resistances at each step and included correction for transverse effects. The equation used to calculate the raw strains before transverse corrections were made was

$$\Delta\varepsilon = \left(\frac{R}{R_o}\right)^{\frac{1}{G.F.}} - 1 , \tag{4}$$

where

$\Delta\varepsilon$ = incremental strain

R = resistance of gage

R_o = resistance of gage at $\Delta\varepsilon = 0$

G.F. = gage factor of the appropriate region.

The total strains are then computed by adding the strain in each region. For instance, the strain in the plastic region of the gage is the sum of the strain at yield plus the incremental strain in the plastic region calculated using the resistance of the gage at yield as R_o in Equation (4). It is assumed that the gage factor changes the same amount whether in compression or tension. Only the sign changes, that is, a decrease for compression, an increase for tension.

Figures 5 and 6 show the volumetric strain vs. true stress for two tests, one in tension, one in compression. These curves compare results with no correction for gage factor and a correction of 0.04.

The correction is seen to be only a few percent of the elastic gage factor but can also be important when measuring cyclic strains. It explains the positive zero shift for cyclic loading as shown by Krempl[11] and Dowling.[12] The apparent strain in the tension part of a cycle is larger than the true value and that in the compression part is smaller than the true value, thus the positive shift in zero. This offset is repeated each cycle so that the cumulative zero shift becomes larger as the number of cycles increases. The total maximum strain difference stays comparatively constant during the first 10-20 cycles since the changes in strain in opposite directions are fairly constant. Subsequently, the gages must not track well. This effect worsens as the strain range increases.

Dowling's[12] cyclic stress-strain curve is also explained by this effect. He obtained smaller compressive strains than tensile strains for the same stress difference.

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[11] *E. Krempl, "Evaluation of High Elongation Foil Strain Gages for Measuring Cyclic Plastic Strains," Experimental Mechanics, 8 (8), 19N-26N, August 1968.*

[12] *N. E. Dowling, "Performance of Metal-Foil Strain Gages During Large Cyclic Strains," Experimental Mechanics, 17, 193-197, May 1977.*

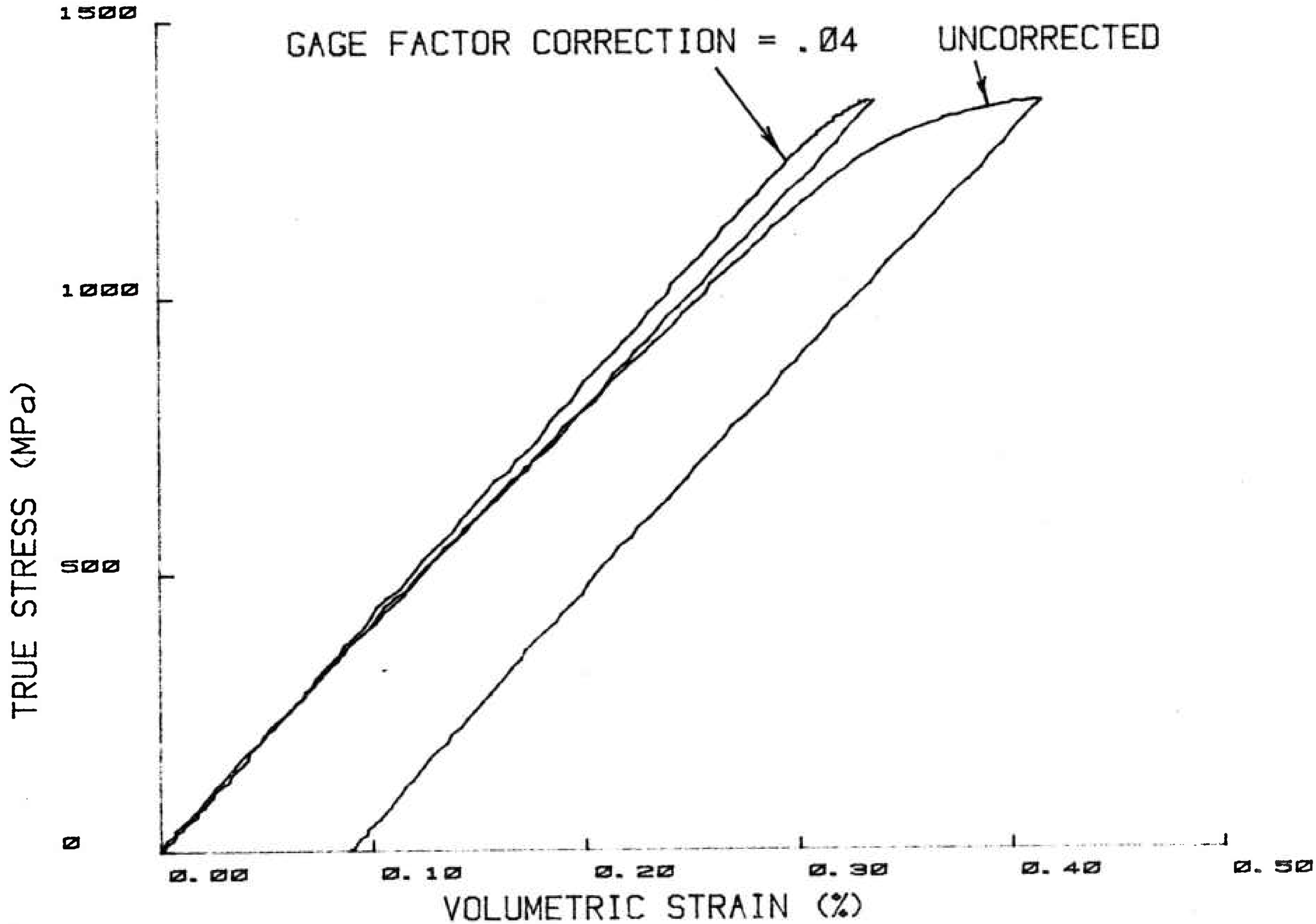

Figure 5. Uniaxial Tension Test on Berylium-Copper.

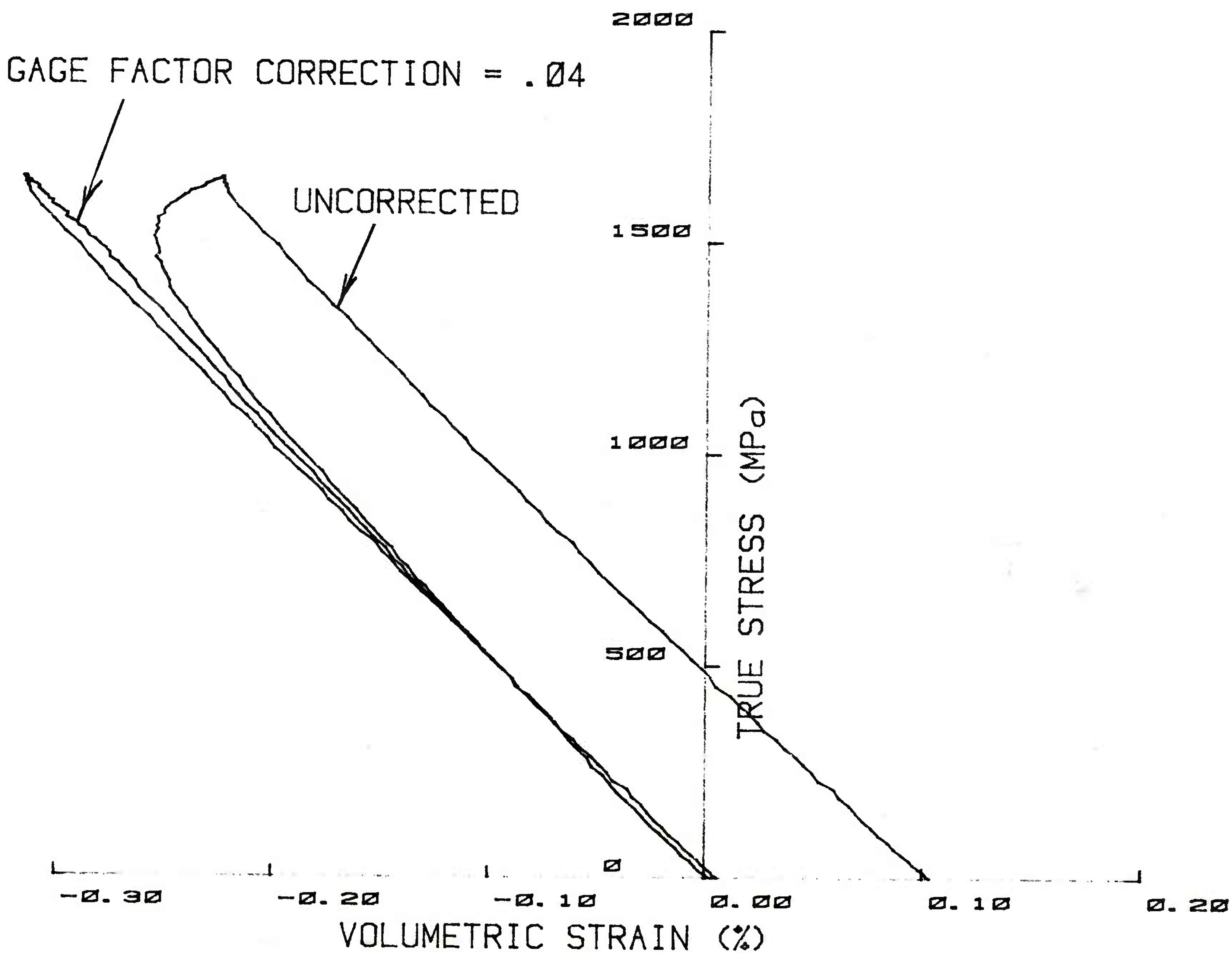

Figure 6. Uniaxial Compression Test on a High-Strength Tool Steel.

ACKNOWLEDGMENTS

The author wishes to thank Mr. Ralph Benck, who performed the density determinations; Mr. Edward Rapacki, who designed the diametral displacement system; and Mr. Dominic Diberardo, whose expert strain gage application was so important to this work.

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12. N. E. Dowling, "Performance of Metal-Foil Strain Gages During Large Cyclic Strains," Experimental Mechanics, 17, 193-197, May 1977.

A-1. P. W. Bridgman, "The Effect of Homogeneous Stress on the Electrical Resistance of Crystals," Physical Review, 42, 858-863, 1932.

A-2. E. Barsis, E. Williams, and C. Skoog, "Piezoresistivity Coefficients in Manganin," Journal of Applied Physics, 41, 5155-5162, 1970.

A-3. D. E. Grady and M. J. Ginsberg, "Piezoresistive Effects in Yterbium Stress Transducers," Journal of Applied Physics, 48, 2179-2181, 1977.

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APPENDIX A

Piezoresistive Response of Strain Gages

APPENDIX A

PIEZORESISTIVE RESPONSE OF STRAIN GAGES

Bridgman[A-1] was the first to formulate the equations necessary to describe the effect of general mechanical stress on the electrical resistivity of crystals. These relationships can be represented through the use of a fourth-rank tensor usually called the piezoresistive tensor.[A-2] For elastic stresses the tensor components or coefficients can be described in terms of strains and a strain tensor defined. This kind of analysis has been used to describe the behavior of various materials and their use in stress[A-2,A-3] and strain gages.[A-4,A-5] For an isotropic material there are only two independent coefficients. Recently Gupta[A-6] has presented an incremental analysis for gage response which uses the piezoresistive tensor combined with an elastic-plastic model for the dimensional deformation of the gage. This model includes the change in resistivity due to plastic work as well as that due to elastic stresses. The analysis is used to describe the response of manganin and yterbium stress gages to one-dimensional shock wave loading but can just as easily be used to analyze strain gage response to tensile or compressive loading.

Gupta[A-6] defines a right-handed coordinate system such that the X axis is along the gage width, Y along the gage thickness, and Z along the gage length. Then the resistance, R_Z, of the gage element with electric field and current density directed along the gage length is derived as

$$\frac{R_Z}{R_o} = [1 + \alpha\ (\Delta\sigma_X + \Delta\sigma_Y + \Delta\sigma_Z) + 2\beta\Delta\sigma_Z + \eta\Delta W^P] \qquad \text{(A-1)}$$

$$* [1 + \Delta\varepsilon_Z] \ / \ [1 + \Delta\varepsilon_X]\ [1 + \Delta\varepsilon_Y] \quad .$$

[A-1] *P. W. Bridgman, "The Effect of Homogeneous Stress on the Electrical Resistance of Crystals," Physical Review, 42, 858-863, 1932.*

[A-2] *E. Barsis, E. Williams, and C. Skoog, "Piezoresistivity Coefficients in Manganin," Journal of Applied Physics, 41, 5155-5162, 1970.*

[A-3] *D. E. Grady and M. J. Ginsberg, "Piezoresistive Effects in Yterbium Stress Transducers," Journal of Applied Physics, 48, 2179-2181, 1977.*

[A-4] *W. P. Mason and R. N. Thurston, "Use of Piezoresistive Materials in Measurement of Displacement Force, and Torque," Journal of the Acoustical Society of America, 29, 1096-1101, 1957.*

[A-5] *W. F. Brace, "Effect of Pressure on Electric-Resistance Strain Gages," Experimental Mechanics, 4, 212-216, July 1964.*

[A-6] *Y. M. Gupta, "Analysis and Modeling of Piezo-Resistance Response," DNA5451F Defense Nuclear Agency, September 1980.*

$\Delta\sigma_X$, $\Delta\sigma_Y$, and $\Delta\sigma_Z$ are changes in applied stress in X, Y, and Z directions. $\Delta\varepsilon_X$, $\Delta\varepsilon_Y$, and $\Delta\varepsilon_Z$ are corresponding changes in strain. α and β are two independent material constants derived from the piezoresistive constants and measured in the elastic range. ΔW^P is the change in plastic work and η a material constant relating changes in plastic work to resistivity. R_o is the resistance of the gage element before deformation.

In order to use Equation (A-1) to analyze a constantan foil strain gage we first have to consider the deformation of the gage with respect to the strain in the underlying substrate and the gage's plastic backing. Brace[A-5] considered this problem for a foil strain gage where the grid length is much larger than the grid width and thickness. He concluded that the foil gage would track the substrate along the gage length but there would be almost no constraint along the width. In other words, the gage would behave as if in uniaxial tension or compression.

Assuming this analysis is correct (other evidence is given in this report), the strain along the gage length, ε_Z, will be equal to the strain of the substrate in that direction. Equation (A-1) can then be written for <u>elastic</u> strains in the gage material

$$\frac{R_Z}{R_o} = [1 + (\alpha + 2\beta)\Delta\sigma_Z]* \frac{[1 + \frac{\Delta\sigma_Z}{E}]}{[1 - \nu \frac{\Delta\sigma_Z}{E}]^2} \tag{A-2}$$

Here $\Delta\sigma_X = \Delta\sigma_Y = \Delta W^P = 0$ and ν and E are Poisson's ratio and Young's Modulus of the gage material.

For small elastic strains the incremental stresses and strains can be replaced by their total values and to first order Equation (A-2) can be written

$$\frac{\Delta R_Z}{R_o} = [(\alpha + 2\beta) + \frac{1 + 2\nu}{E}]\ \sigma_Z \tag{A-3}$$

or

$$\frac{\Delta R_Z}{R_o} = [(\alpha + 2\beta)E + 1 + 2\nu]\ \varepsilon_Z \tag{A-4}$$

Equation (A-4) is recognized as the usual gage factor equation[10] with $(\alpha + \beta)E$ equal to the strain coefficient of resistivity.

The constants α and β are usually obtained from two experiments; the change in resistance due to hydrostatic pressure and the change due to uniaxial tension.[A-1,A-3,A-5] In the case of an isotropic strain gage material, only the tension experiment is needed if the elastic mechanical properties are known. It is interesting to point out that if a material has a gage factor of 2.0,

the piezoresistive coefficient, $\alpha + 2\beta$, is numerically equal to its linear compressibility, $K_\ell = \frac{1 - 2\nu}{E}$. When the gage material becomes plastic since $\Delta\sigma_Z$ is zero or comparatively small and $\nu = 0.5$, the plastic gage factor is also 2.0. Arit[A-7] has used a different analysis to point this out. In this case, as was stated previously, the plastic gage factor is actually $2.0 + \varepsilon$, since no plastic volume change is postulated. For the gage factor to be $2.0 + \varepsilon$ it is only necessary that the resistivity per unit volume stay constant. This, of course, implies that in Equation (A-1), $\eta = 0$.

The gages studied in this report did show gage constant changes when the gage material yielded. This implies that $\eta \neq 0$. The value of η can be estimated from the change in gage factor if a uniaxial stress-strain curve for the gage material is known. Appendix B describes some experiments on constantan wires which measured both uniaxial stress-strain curves and gage factors in the elastic and plastic regions and gives an estimate of η.

[A-7] *G. Arit, "The Sensitivity of Strain Gauges," Journal of Applied Physics, 49, 4273-4274, 1978.*

APPENDIX B

Constantan Wire Experiments

APPENDIX B

CONSTANTAN WIRE EXPERIMENTS

Two types of experiments were performed on constantan wires. One measured the load-resistance properties in tension, and one measured the gage factor directly.

The experiments were performed with an Instron Testing Machine. The load-resistance tests were made with 0.32 mm diameter wire approximately 0.58 m long. The wires were mounted in the machine with insulated clamps. Heavier constantan wire leads were soldered to the wire near the top and bottom clamps. These leads were brought to the terminals of a 6 1/2 digit digital multimeter with a four terminal resistance measuring circuit. This procedure kept thermal e.m.f.'s at a minimum. A calibrated load cell was used to measure the load. The crosshead was run at 1.0 mm per minute and load-resistance data obtained.

The Instron Machine has calibrated screws which move the crosshead at constant speed. If the original wire length is known, increases in length after specified times can be used to calculate strains from the known cross-head speed. It was not possible to measure both load and strain in the same experiment because the load cell used had too high a compliance and extended with load. Therefore, the load cell was removed for the strain-resistance experiments.

In order to increase the accuracy of the measurement, a jig was made to increase the length of the wire used. This consisted of a sturdy aluminum pipe 50 mm in diameter with 13 mm wall thickness. An insulated clamp was attached to the top of the pipe which was mounted over the hole in the top stationary crosshead with the load cell removed. This increased the original distance between clamps to approximately 1.6 m. The total resistance was doubled by using a small stiff wire loop at the bottom clamp which was attached to the moving crosshead. The wire was run from the top clamp down through the loop and back up to the top. The crosshead was usually operated at a speed of 1.0 mm per minute. Some wires were stretched continuously to approximately 2% strain. Some wires were loaded and unloaded up to five times with total strain to 3%.

This procedure gave length-resistance data for elastic loading, elastic-plastic loading, and unloading. Figure B-1 shows some results. The data was reduced by fitting a least-squares straight line to the natural logarithm of the two measurements, that is, $\ln \frac{\ell}{\ell_o}$ and $\ln \frac{R}{R_o}$. Some of the data points in the vicinity of the change in loading direction were not used. The slope of the lines give the gage factor for each of the loading sequences. Unloading is shown with positive strain to avoid confusion. Figure B-2 shows an engineering stress-strain curve obtained from the data of the two types of experiments.

In Figure B-1 note that when plastic deformation occurs the gage factor increases and during unloading it changes again. With this data, we can calculate the piezoresistive coefficient for uniaxial stress and also estimate the value of η in Equation (A-1).

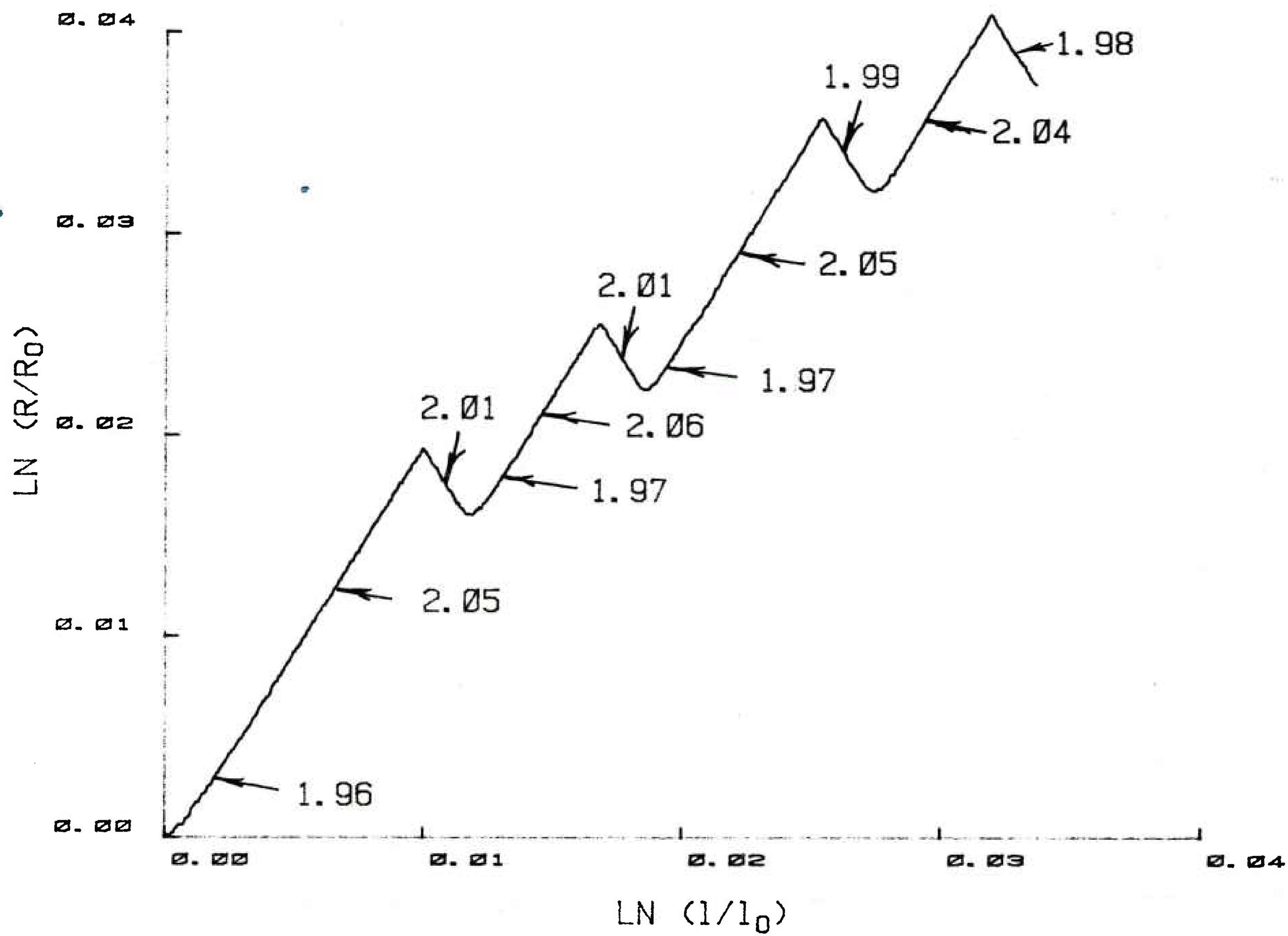

Figure B-1. Uniaxial Tension Test on Constantan Wire. Numbers Designate Slopes of the Lines or Gage Factors.

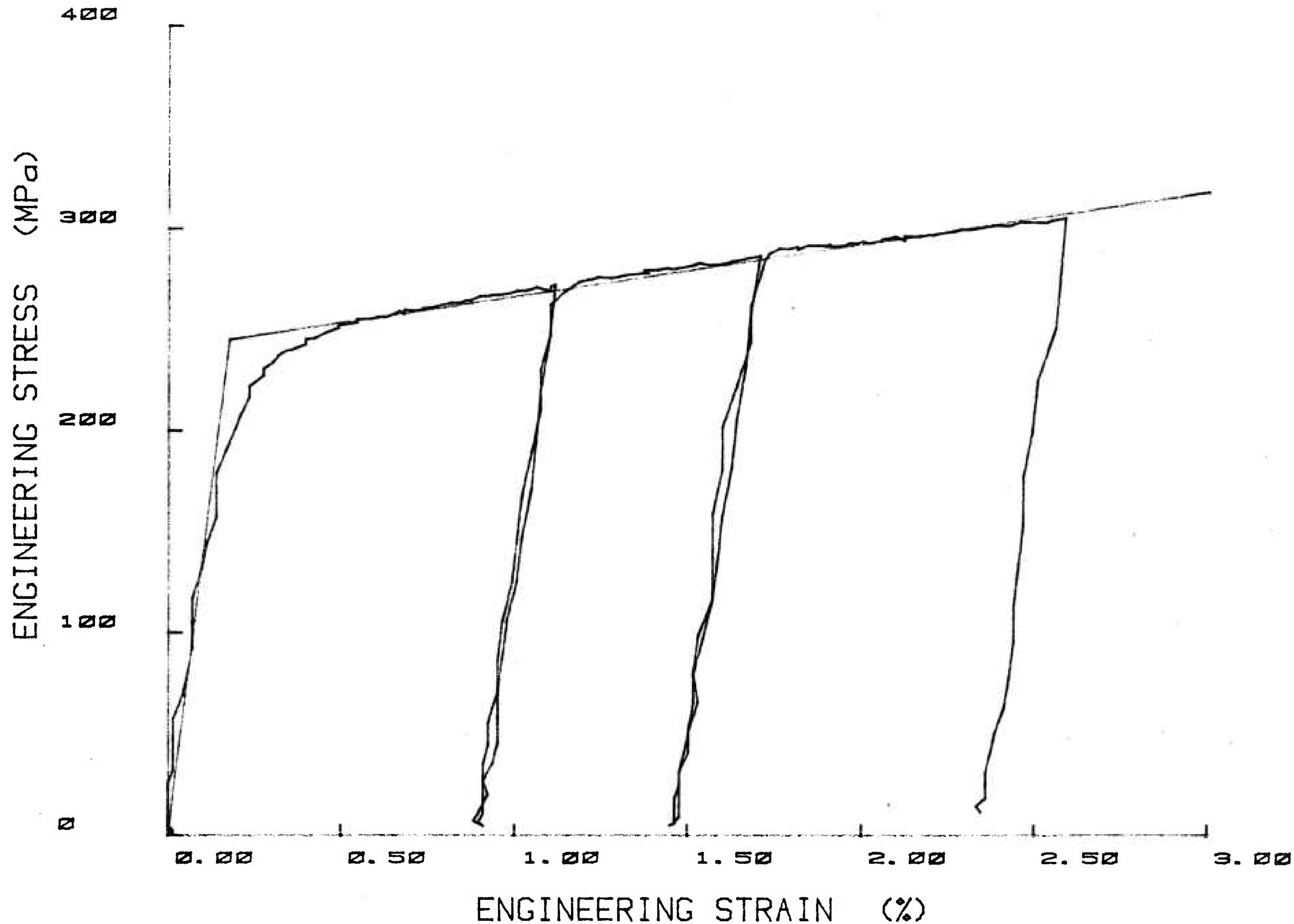

Figure B-2. Uniaxial Tension Test on Constantan Wire. Straight Lines are Bilinear Approximation of Stress-Strain Curve.

The stress-strain data were approximated by a bilinear curve as seen in Figure B-2. The equations for this curve are

$$\begin{aligned} &\text{Elastic } \sigma = 1.40 \times 10^{5}\varepsilon \;;\; 0 < \varepsilon < .0017 \\ &\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{MPa.} \qquad\qquad (B\text{-}1) \\ &\text{Plastic } \sigma = 238 + 2570\,(\varepsilon - .0017);\; \varepsilon > .0017 \end{aligned}$$

The elastic gage factor of the wires was measured as 1.99 from the average of 6 tests with 12 loadings and unloadings. Using 2.15×10^{-6} MPa^{-1} as the linear compressibility of constantan,[B-1] Poisson's ratio was calculated as 0.35, which corresponds to the value used by Brace.[A-5] Then from Equation (A-4) the piezoresistive coefficient, $\alpha + 2\beta$, has a value 2.1×10^{-6} MPa^{-1}.

The plastic gage factor, taken as an average from the same tests, was 2.05, determining the plastic work with use of Equation (B-1). The value of η in Equation (A-1) is calculated to be approximately 2×10^{-4} MPa^{-1}.

[B-1] *P. W. Bridgman, "Effects of Pressure on Binary Alloys," Proceedings of American Academy, 84, 131-177 (1957).*

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